 Emergence: Complexity and Organization.

More lessons from complexity

The origin - The root of peace

September 30, 2006 · Forum

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Puente C. More lessons from complexity: The origin - The root of peace. Emergence: Complexity and Organization. 2006 Sep 30 [last modified: 2016 Nov 26]. Edition 1. doi: 10.emerg/10.17357.03cb3538f3ddfc88d1efff684815f4be.

Abstract

The last few decades have witnessed the development of a host of ideas aimed at understanding and predicting nature's ever present complexity (see for instance, Mandlebrot, 1982; Bak, 1996; Wolfram, 2002). It is shown that such a work provides, through its detailed study of order and disorder, a suitable framework for visualizing the dynamics and consequences of mankind's ever present divisive traits. Specifically, this work explains how recent universal results pertaining to the transition from order to chaos via a cascade of bifurcations point us to a serene state, symbolized by the convergence to the origin in the root of a Feigenbaum's tree, in which we all may achieve peace.

The Logistic Map

Recall the exotic dynamics of the *logistic map* (see for example, Rasband, 1990; Peitgen, *et al*., 1992; Beck & Schlögl, 1993):

$X_{k+1} = a\, X_k \cdot (1 – Xk)$

that is, the chain of ultimately stable (and unstable) values $X_8(a)$ found iterating the map, where X_ks denotes the normalized size of a *population* at generation *k* and *a* is a free parameter having values between 0 and 4:

When *a* = 1, the logistic *parabola* is below the one to one line (added to aid in the calculations), and then $X_8 = 0$ (Figure 1);

When 1 < *a* = 3, the parabola is above the line **X = Y** and $X_8 = (a – 1)/a$, the non-zero intersection between the curve and the straight line (Figure 2);

When 3 < *a* = 3.449…, $X_8 = \{X_8(1), X_8(2)\}$ and the population settles into an oscillation repeating every two generations (Figure 3);

When 3.449… < *a* = 3.544…, $X_8 = \{X_8(^3), X8(^4), X_8(^5), X_8(^6)\}$. The population ultimately repeats every four generations, and the dynamics have experienced a bifurcation (Figure 4);

When *a* is increased up to a value a_8= 3.5699…, successive bifurcations in powers of two happen quickly, that is, the dynamics repeat exactly every 2*n* generations, for any value of n;

When $a_8 < a = 4$, behavior is found either periodic or non periodic. For instance, for *a* = 3.6 an infinite *strange attractor* with a whole in the middle is found (Figure 5);

When *a* = 3.83, $X_8 = \{X_8(^1), X_8(^2), X_8(^3)\}$ and the dynamics oscillate every 3 generations (Figure 6);

When *a* = 4, the most common behavior is non periodic and a dense strange attractor over the interval [0, 1] is found (Figure 7).

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Fig. 1: Convergence to the origin

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Fig. 2: Convergence to a fixed point

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Fig. 3: Convergence to a 2-cycle.

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Fig. 4: Convergence to a 4-cycle

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Fig. 5: Convergence to a "dusty" non-repetitive attractor

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Fig. 6: Convergence to a 3-cycle

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Fig. 7: Convergence to a maximal non-repetitive set

At the end, the cascade of stable *period-doubling bifurcations* (before a_8) and the emergence of chaos (strange attractors) intertwined with periodic behavior (including any period greater than two) is summarized via the celebrated **Feigenbaum's diagram** (Figure 8).

This is so named after Mitchell Feigenbaum who showed that the bifurcation openings and their durations happen **universally** for a class of unimodal maps according to two universal constants $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$, as follows (Feigenbaum, 1978) (refer to Figure 9):

d_n/d_{n+1} ? $F_1 = -2.5029\ldots$, $?_n/?_{n+1}$? $F_2 = 4.6692\ldots$

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Fig. 8: Bifurcations tree for the logistic map

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Fig. 9: Successive bifurcations

x? corresponds to supercycles, i.e., ½ for the logistic map

For example, other "*fig trees*" guided by $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ and for the two simple mappings $f(X) = a\,X \cdot (1 — X^3)$ and $f(X) = a\,X \cdot (1 – X)^3$ are shown below[1]. Notice how such contain: a straight "*root*," a bent "*branch*," bifurcation branches, and then, in an orderly intertwined fashion, following Sharkovskii's order (see for example, Rasband, 1990; Peitgen, *et al*., 1992; Beck & Schlögl, 1993), periodic branches, and the ever dusty "*foliage of chaos*," where the unforgiving condition of sensitivity to initial conditions rules.

Chaos theory and our quest for peace

As the dynamics of the logistic map describe several physical processes (see for instance, Cvitanovic, 1989; Bai-Lin. 1984), including fluid *turbulence* as induced by heating, that is, convection, it is pertinent to consider such a *simple* and *universal* mechanism to study how “chaos” and its related condition of “violence” may arise in the world.

Given that the key parameter a, associated with the amount of heat (Libchaber & Maurer, 1978), dictates the ultimate organization of the fluid, we may see that it is wise to keep it small (in the world, and within each one of us) in order to avoid undesirable “nonlinearities.” For although the allegorical fig trees exhibit clear order in their pathway towards disorder, we may appreciate in the uneasy jumping on strange attractors (and also on periodic ones), the anxious and foolish frustration we often experience (so many times deterministically!) when we, by choosing to live in a hurry, travel from place to place to place in “*high heat*” without finding our “*root*.”

In this spirit, the best solution for each one of us is to slow down altogether the pace of life, coming down the tree, so that by not crossing the main *threshold* **X** = **Y**, that is, by choosing a = 1, we may surely live without turbulence and chaos in the robust state symbolized by X8 = 0[2,3]. For there is a marked difference between a seemingly laminar condition as it happens through tangent bifurcations (see for example, Rasband, 1990; Peitgen, et al., 1992; Beck & Schlögl, 1993) and being truly at peace, for the former invariably contains dramatic bursts of chaos and ample intermittency (see for example, Rasband, 1990; Peitgen, et al., 1992; Beck & Schlögl, 1993).

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Fig. 10: Bifurcation diagrams for two simple nonlinear maps

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Fig. 11: Orbits of the logistic map (a = 4) ending up at zero

As **zero**, that is, converging to the origin, is identified as the desired state, it is sensible to realize that such an organization, a *trivial* solution for X_8(a), even if unstable, may be reached even when the worst chaos engulfs us (a = 4). For the precise dynamics of the *pre-images of zero* do not wander for ever in high heat, but rather find the way to the **root** through a delicate hopscotch by the *middle*[4] (Figure 11). For it is tragic indeed to “oscillate for ever” (Figure 12). And more tragic yet to be close to “*the point*” and miss it altogether forever[5] (Figure 13). For the *butterfly effect*, with all probability and contrary to the illusion that it provides us with options, leaves us irremediably trapped in dust.

At the end, the emergence of the modern science of complexity helps us visualize our ancient choices. It is indeed best for us to live in serenity and in a **simple** manner, not amplifying and hence heeding the voice. For only the conscious order of **Love** does not suffer the destiny of arrogant stubbornness that justly receives the same “bad luck” of a parabolic tree that did not have any fruit, the same one that with its tender branch(es) and budding leaves, also announces horrendous times, but also very good ones, times of **joy** and of **friendship**.

Acknowledgements

This piece is dedicated, in memoriam, to Fr. Rafael García Herreros, a humble man with a clear vision, whose special touch instilled in me the need to dream, in order to contribute to mend our chaotic world. Figure 12 Some orbits of the logistic map (? = 4) leading to a 3-cycle Figure 13 Some orbits of the logistic map (? = 4) ending in a strange attractor Some orbits of the logistic map (? = 4) leading to a 3-cycle Some orbits of the logistic map (? = 4) ending in a strange attractor (Carlos E. Puente) In the confines of science majestically stands a tree, with all numerals in dance in emergent chaos to see. In the instance of a trance a good day I drew a link, and here it is, at a glance, the wisdom that I received. Foliage of disorder trapped in empty dust, jumps astir forever enduring subtle thrust. Crossing of the outset leaving faithful root, looming tender offset failing to yield fruit. Cascade of bifurcations, increasing heat within, inescapable succession of branches bent by wind. Sprouting of dynamics attracted to the strange, oh infinity reminding at the origin: the flame. In the midst of chaos there is a small gate leading to fine rest. In the midst of chaos there are loyal paths inviting to a dance. On top of the fig tree there is a key point that runs to the core. On top of the fig tree there is a clear light that averts a fright. In the midst of chaos there is leaping game discerning the way. In the midst of chaos there is a fine well watering the brain. On top of the fig tree there is a clean frame that cancels the blame. On top of the fig tree there is mighty help that shelters from hell. In the midst of chaos, look it is there, in the midst of chaos, logistics in truth, in the midst of chaos, a clear faithful route, in the midst of chaos, leading to the root. On top of the fig tree, this is no delusion, on top of the fig tree, a sought needle’s eye, on top of the fig tree, the symbol of wheat, on top of the fig tree, surrounded by weeds. Could it be, oh my friends, that science provides a rhyme?, for a rotten tree foretells the very advent of time. Could it be, oh how plain, that nature extends a call?, for old parable proclaims the crux in growing small.

References

1. Bai-Lin, H. (1984). Chaos, World Scientific, ISBN 9971966506.

2. Bak, P. (1996). How Nature Works, Copernicus, ISBN 038798738X (1999).

3. Beck, C. and Schlögl, F. (1993). Thermodynamics of Chaotic Systems: An Introduction, Cambridge, UK: Cambridge University Press, ISBN 0521484510 (1995).

4. Cvitanovic, P. (ed.) (1989). Universality in Chaos, Adam Hilger, ISBN 0852742606.

5. Feigenbaum, M. J. (1978). "Quantitative universality for a class of nonlinear transformations," Journal of Statistical Physics, ISSN 0022-4715, 19(1): 25.

6. Libchaber, A. and Maurer, J. (1978). "Local probe in a Rayleigh-Benard experiment in liquid helium," J. de Phys. Lett. 39, L369.

7. Mandelbrot, B. B. (1982). The Fractal Geometry of Nature, Freeman, ISBN 0716711869.

8. Peitgen, H.-O., Jürgens, H., Saupe, D. (1992). Chaos and Fractals: New Frontiers of Science, New York, NY: Springer-Verlag, ISBN 0387202293 (2004).

9. Puente, C. E. (2006a). "Lessons from complexity: The hypotenuse - The pathway of peace," Emergence: Complexity & Organization, ISSN 1521-3250, 8(2): 96-101.

10. Puente, C. E. (2006b). The Hypotenuse: An Illustrated Scientific Parable for Turbulent Times, AuthorHouse, ISBN 1425901743.

11. Rasband, S. N. (1990). Chaotic Dynamics of Nonlinear Systems, Chichester, UK: John Wiley & Sons, ISBN 0471184349 (1997).

12. Wolfram, S. (2002). A New Kind of Science, Wolfram Media Inc., ISBN 1579550088.