Heterogeneity of degree and the emergence of cooperation in complex social networks

December 31, 2008 · Academic
Gregory Todd Jones, Gregory Jones
Jones GT, Jones G. Heterogeneity of degree and the emergence of cooperation in complex social networks. Emergence: Complexity and Organization. 2008 Dec 31 [last modified: 2016 Nov 30]. Edition 1. doi: 10.emerg/10.17357.8fd1329f69a8f0b6adbebaf52f783001.

Abstract

The evolution of cooperative, pro-social behavior under circumstances in which individual interests are at odds with common interests—circumstances characterized as social dilemmas—remains a largely unsolved, multidisciplinary puzzle. Approaches to these types of problems have, for the most part, been applications of evolutionary game theory. While the study of networks, complex systems, and nonlinear dynamics has pervaded most scientific disciplines, the application of related tools to the study of social dilemmas represents a very new, but extremely promising means of shedding light on the quandary of cooperation. In this work, we situate agents engaging in social dilemma games on complex social networks, allowing us to more fully investigate the impact of average degree and degree variance, or heterogeneity of degree, on the evolution of pro-social behavior. Our results suggest that increasing homogeneity of degree produces network effects that make the emergence of pro-social behaviors more likely thereby increasing overall social welfare. As such, homogeneity of degree is properly thought of as a collective good.

Introduction

Cooperation has been vital to the evolution of all living things, including single-celled organisms (Velicer, 2005, 2003; Velicer & Stredwick, 2002; Crespi, 2001; Velicer *et al.*, 2000; Boorman & Levitt, 1980), fish (Brosnan *et al.*, 2003; Dugatkin, 1991, 1992, 1997; Milinski, 1987), birds (Brown & Brown, 1996; Faaborg *et al.*, 1995), canines (Creel & Creel, 2002; Courchamp & Macdonald, 2001; Fentress & Ryon, 1986), felines (Caro, 1994; Packer & Pusey, 1982), non-human primates (Brosnan & de Waal, 2003; de Waal, 1996, 1982; Harcourt & de Waal, 1992; Chapais, 1992), and humans (Ostrom *et al.*, 1999; Fehr & Fischbacher, 2003; Johnson *et al.*, 2003).

Even so, the evolution of cooperative, pro-social behavior under circumstances in which individual interests are at odds with common interests, (circumstances characterized as social dilemmas (Gotts *et al.*, 2003; Dawes & Messick, 2000)), remains a largely unsolved, multidisciplinary puzzle (Hammerstein, 2003). Approaches to these types of problems have, for the most part, been applications of evolutionary game theory (Gintis, 2000; Hofbauer & Sigmund, 1998; Maynard-Smith, 1982; Maynard-Smith & Price, 1973; Trivers, 1971; Hamilton, 1967; von Neumann & Morgenstern, 1944) and due to their importance as generalized models of many important socio-economic situations (Tomassini, 2006), iconic games such as the Prisoner’s Dilemma have been widely employed as metaphors (Doebeli & Hauert, 2005; Axelrod & Hamilton, 1981; Axelrod, 1984; Nowak & Sigmund, 1992, 2004; Nowak & May, 1992; Maynard-Smith, 1982; Sugden, 1986).

At the same time, the study of networks, complex systems, and nonlinear dynamics has pervaded all of science (Strogatz, 2001)[1]. Indeed, E.O. Wilson, who once characterized the evolution of cooperation as one of the greatest challenges for modern biology (Wilson, 2000), more recently made a more emphatic appeal for research on complex systems. “The greatest challenge today, not just in cell biology and ecology, but in all of science, is the accurate and complete description of complex systems. Scientists have broken down many kinds of systems. They think they know most of the elements and forces.

Image not found
https://journal.emergentpublications.com/wp-content/uploads/2015/11/cd16d7bf-4422-dd11-0926-ded3be1f779f.png

Fig. 1: Figure 1

A complex network in which each agent has N — 1 possible connections, a full range of degree and heterogeneity of degree.

The next task is to reassemble them, at least in mathematical models that capture the key properties of the entire ensembles.” (Wilson, 1998: 85). The application of complex systems tools and network analysis methodologies to the study of social dilemmas represents a very new, but extremely promising means of shedding light on the quandary of cooperation (Hanaki *et al*., 2007; Fu *et al*., 2007; Ohtsuki *et al*., 2006; Santos *et al*., 2006a, 2006b; Szabo & Fath, 2006; Tomassini *et al*., 2006; Santos & Pacheco, 2005; Santos *et al*., 2005; Lieberman *et al*., 2005; Abramson & Kuperman, 2001; Skyrms & Pemantle, 2000).

In this work, we situate players engaging in social dilemma games on complex social networks, thereby avoiding artificial limitations placed on the number of partners each player may have (a measure known as “degree”) by the more structured, lattice-like architectures common to the earlier literature on such games. These more generalized network architectures offer a full range of degree (from 0 to N — 1), which allows us to more fully investigate the impact of average degree on the evolution of pro-social behavior. In addition, large numbers of simulations subjected to statistical analysis help to address the primary concerns of this research: the effect of degree variance, what we call *heterogeneity of degree*, on cooperative behavior and the extent to which this effect can be separated from other explanatory factors. We begin with detailed descriptions of the models under study. Next, we describe results of simulations conducted in this framework, systematically varying certain aspects of network architecture and measuring the effect on the evolution of cooperative behavior. Finally, we discuss the implications of these findings for conflict management and briefly describe two examples where they may find practical application.

Model Descriptions

We examine a population of N players each engaging in a repeated prisoner’s dilemma game with a neighborhood of other players defined by particular network architectures. See Figure Two. The set of players with whom player *i* interacts in period *t* is denoted by $?_{i,t}$. In each generation, which is comprised of *g* games, each player accumulates an adaptive score based upon a standard payoff matrix described in more detail below. At the end of each generation, each player observes the payoffs and strategies of each neighbor and stochastically updates[2] their strategy with probability *?* ? [0,1] by imitating

Image not found
https://journal.emergentpublications.com/wp-content/uploads/2015/11/baf8a1ea-bfb8-34b4-d482-53b3ea412d03.png

Fig. 2: Figure 2

Schematic illustration of various network architectures, all with 25 nodes, roughly in ascending order of heterogeneity. (a) Fully connected network. (b) Ring lattice with all nodes connected to it neighbors out to some range k (here k = 3). (c) Small world network starting with ring lattice and adding shortcut links between random pairs of nodes. (d) Random network constructed with connection probability, p = .15. (e) Scale-free network constructed by attaching nodes at random to previously existing nodes, where the probability of attachment is proportional to the degree of the target node, i.e., "the rich get richer."

the strategy of the neighbor with the highest adaptive score (including themselves). Ties in high scores are broken at random.

Strategic Dynamics

For each period *t*, players choose to either cooperate *C* or defect *D* with each of its neighbors $?_{i,t}$ and the strategic decision for each neighbor is independent of the decisions with regard to other neighbors, that is, a player can choose to cooperate with some neighbors and defect with others (c.f. Hanaki, 2007). Each neighbor *j* ? $?_{i,t}$ faces a symmetrical decision giving rise to the to a standard payoff matrix

i, j	C	D
C	R, R	S, T
D	T, S	P,P

Where $?(s_i, s_j)$ is the payoff for player *i* choosing strategy s_i when neighbor *j* chooses strategy s_j,

$?(C,C) = R$, $?(C,D) = S$, $?(D,C) = T$

and $?(D,D) = P$.

In keeping with the standard structure of a social dilemma, $T > R > P > S$, which makes defection a dominant strategy, that is, defection results in a higher payoff as compared to cooperation regardless of what strategy the opponent neighbor chooses, and $2R > (T + S)$, which insures that mutual cooperation is preferred over all other strategy sets in the sense that it produces maximum aggregate outcomes. The unique equilibrium for the game, mutual defection, thus leads to a Pareto-suboptimal solution.

For each generation, each player accumulates an adaptive score for g games for all neighbors. Following the logic that the maintenance of networks with more neighbors would involve more cost than networks with fewer neighbors, we reduce adaptive scores by $?(k)$, the total cost of interaction with a network of *k* neighbors. Thus, the net payoff for each player *i* accumulated in a time period *t* is:

Image not readable or empty
https://chart.googleapis.com/chart?cht=tx&chf=bg,s,FFFFFF00&chco=000000&chl=%5C%5B%7B%5CPi%20_%7Bi%2Ct%7D%7D%20%3D%20%5Csum%5Climits_%7Bj%20

where *?(k)* is an increasing function of *k* with the specific form *?(k)* = $ck^{?}$, where *?*?1 and 0 ? *c* ? *P* (Hanaki, 2007).

Imitation Dynamic

After each generation, each player examines the accumulated adaptive scores of each of its neighbors, $?_{i,t}$, and its own accumulated adaptive score, and either adopts by imitation the strategy of the most successful neighbor, or keeps it own strategy if it has been most successful, to be employed in the next generation, formally

Image not readable or empty
https://chart.googleapis.com/chart?cht=tx&chf=bg,s,FFFFFF00&chco=000000&chl=%5C%5B%7Bs_%7Bi%2Ct%20%2B%201%7D%7D%20%3D%7B%5Carg%20%5Cmax%20

Image not found
https://journal.emergentpublications.com/wp-content/uploads/2015/11/fb85623d-ed68-1e74-5863-66712b9270b7.png

Fig. 3: Table 1

Model Variable Specification

If more than one player in the neighborhood shares the highest accumulated adaptive score, ties are broken at random.

Network measures

For each run of the simulation, which is comprised of a large number of generations sufficient to arrive at equilibrium in the strategy population, a number of variables are recorded: population, average degree, heterogeneity of degree, network architecture, and cooperation. Network architecture is recorded as lattice, small world, random, or scale-free (fully connected is a special case of lattice). Cooperation is measured as a ratio of player decisions to cooperate to the total number of cooperation/defection decisions. See Table 1.

Simulation Results

We ran the simulation as described above 1,000 times creating stochastic networks by drawing network architecture uniformly from lattice, small world, random, or scale-free; drawing population uniformly from a range of 10 to 100; and drawing average degree uniformly from a range of 2 to 10. Heterogeneity of degree ranged from 0 to 4 as a function largely of network architecture. Each run was for 1,000 generations with the cooperation ratio measured in the last 100.

First, in Model 1, we regressed cooperation on population, average degree, heterogeneity of degree and three indicator variables representing four network architectures, lattice (as the base case), small world, random and scale free. We included the indicator variables to capture any variation resulting from network architectural differences not captured by the other independent variables.

We hypothesized that population size would have a positive effect on cooperation, and that both average degree and heterogeneity of degree would have a negative effect. While Model 1 bore out the first two hypotheses (population coefficient = .245, $p <= .000$ and average degree coefficient = -8.170, $p <= .000$), heterogeneity of degree showed a significant positive effect (heterogeneity of degree coefficient = 31.847, $p <= .000$). However, collinearity diagnostics indicated that the two most heterogeneous network architectures were highly collinear with the heterogeneity

Image not found https://journal.emergentpublications.com/wp-content/uploads/2015/11/bf7956d4-8983-ecb3-60a8-813a1b25b5cb.png

Fig. 4: Model 1

Cooperation (“Coop”) regressed on Population (“Pop”), Average Degree (“Degree”), Heterogeneity of Degree (“Hetero”) and three indicator variables representing four network architectures, Lattice (base case), Small World (“SWDum”), Random (“RNDum”), and Scale Free (“SFDum”).

Image not found https://journal.emergentpublications.com/wp-content/uploads/2015/11/d3cebb2f-b2da-85c2-ee41-d42ff89b7b28.png

Fig. 5: Model 2

Cooperation (“Coop”) regressed on Population (“Pop”), Average Degree (“Degree”), and Heterogeneity of Degree (“Hetero”)

of degree variable (variance proportions on dimension 7: random network = .91, scale-free network = .94, and heterogeneity of degree = .91). This collinearity made model coefficients uninterpretable.[3] Further, the small-world network indicator variable failed to achieve statistical significance (p = .660). These results offered confidence that the indicator variables were not adding significant additional explanatory power.

Subsequently, in Model 2, we removed the indicator variables, and regressed cooperation on population, average degree, and heterogeneity of degree. In this more parsimonious model, population had a significant positive effect (population coefficient = .428, p <= .000), average degree had a significant negative effect (average degree coefficient = -4.78, p <= .000), and heterogeneity of degree had a significant negative effect (heterogeneity of degree coefficient = -4.208, p <= .001). Additionally, collinearity diagnostics showed that each of the independent variables was loading highly on its own dimension (population variance proportion on dimension 4 = .75, average degree variance proportion on dimension 3 = .86, heterogeneity of degree variance proportion on dimension 2 = .93). Based on this evidence, we concluded that heterogeneity of degree has a significant *negative* effect on the evolution of cooperation and that this effect is *independent* of the negative effect of average degree.

Discussion

Generalized models of social dilemmas have been employed to increase understanding related to many important socio-economic circumstances (Tomassini, 2006), providing insights into the importance of social architecture that may even dominate individual preference (Schelling, 1978) and the ability of local interaction or “correlated association” to drive cooperative outcomes contrary to those predicted by Nash equilibria (Skyrms, 1996, 2003). These insights have been shown to have a high level of universality, with significance for numerous human social contexts as well as many non-human social species.

Our work places these social dilemma dynamics on complex social networks and confirms the importance of local interaction. As average degree increases and the relative importance of correlated association is decreased, our models demonstrate that pro-social behavior is less likely to evolve. Our models also offer a new insight. As the variance associated with degree within the network, or the heterogeneity of degree increases, the likelihood of pro-social strategy evolution is again reduced, and this negative effect is shown to be independent of the negative effect of average degree.

In *Bowling Alone*, Robert Putnam worries that the decline of social capital that he sees in the declining memberships in civic organizations may undermine the civil engagement that according to him is necessary for a strong democracy (Putnam, 2001). The results of our study suggest that the problem may be more nuanced. It may not be, in fact, the mere magnitude of social connections, but the nature of these connections that should concern us most.[4] Merely promoting the development of dense social networks may lead us down a path to social decline. More important may be the design of institutions that promote homogeneity in social connectedness — increasing homogeneity of degree produces network effects that make the emergence of pro-social behaviors more likely thereby increasing overall social welfare. As such, homogeneity of degree is properly thought of as a collective good.

This prescription offers those involved in conflict management a deceptively simple means of promoting the evolution of cooperation within any given group — by increasing homogeneity in social connectedness. Astute readers may point out, with appropriate concern, that such homogeneity can be obtained by suppressing highly connected individuals as well as taking steps to increase connectedness among relative social isolates. To this objection we would respond by suggesting that cooperation is certainly not the only goal for would-be social engineers and it is, in our view, not the most important goal. Freedom of association may trump cooperation in these considerations and counsel against attempting to restrict the density of any individual’s social capital. On the other hand, helping to integrate social isolates into beneficial social networks would, if we are right, benefit us all. The European Union has made more progress along these lines. Where as in the United States, we largely treat poverty as a welfare problem and provide financial resources, the EU views poverty as a lack of access to social capital and offers targeted transportation, Internet access, and job training and education — all directed at social inclusion.

One other example, that offers a living laboratory of sorts where social architecture could be quite easily manipulated, are the plethora of on-line social network communities that have sprung up on the Internet in recent years. In these virtual communities, membership categories and differential incentive structures could allow operators to experimentally influence the structure of networks and sub-networks and facilitate the capture of metrics for cooperation. Such experiments would not only help to refine interventions that conflict managers may employ in the “real” world, but would increasingly find direct application as the “real” world and the virtual world of social interaction become less and less distinguishable.

Notes

1.
For example, these ideas have recently been employed to study the architecture of the Internet (Faloutsos *et al*., 1999), the topology of food webs (Williams & Martinez, 2000), and the metabolic network of the bacterium Escherichia coli (Jeong *et al*., 2000).

2.
Note that where ?<1, updates are asynchronous (Huberman & Glance, 1993).

3.
This is not to say that Model 1 as a whole is unreliable. Highly correlated independent variables can bring about dramatic, unpredictable changes in coefficient estimates, sometimes even changing the direction, or sign. Additionally, in this particular case, the collinearity of two of the indicator variables with the heterogeneity variable and the lack of significance related to the third indicator variable suggests that these indicator variables do not explain enough additional variance to warrant their inclusion.

4.
Putnam's work also suggests that local cohesiveness, or "clumpiness" may have a determinative effect—this is a network measure not included here, but planned for future studies.

Acknowledgements

This research was partially funded by a generous grant from the William and Flora Hewlett Foundation. Much of this work was completed while a visitor at the Max Planck Institute for Research on Collective Goods in Bonn, Germany. Their gracious hospitality is gratefully acknowledged. The author acknowledges helpful comments and suggestions from colleagues at the 2007 NECSI Winter School on Complex Systems at MIT, the Society for the Evolutionary Analysis of Law, the Gruter Institute for Law and Behavioral Research, the North American Association for Computational Social and Organization Sciences, and the participants at the Georgia State University Law Review's 2008 Symposium on Dynamical Jurisprudence.

References

- Abramson, G. and Kuperman, M. (2001). "Social games in a social network," Physical Review E - Statistical, Nonlinear and Soft Matter Physics, ISSN 1539-3755. 63: 030901.
- Axelrod, R. (1984). The Evolution of Cooperation, ISBN 9780465021215.
- Axelrod, R. and Hamilton, W. (1981). "The evolution of cooperation," Science, ISSN 0036-8075, 211: 1390-1396.
- Boorman, S.A. and Levitt, P.R. (1980). The Genetics of Altruism, ISBN 9780121156503.
- Brown, C.R. and Brown, M.B. (1996). Coloniality in the CliffSwallow: The Effect of Group Size on Social Behavior, ISBN 9780226076263.
- Brosnan, S.F. and de Waal, F.B.M. (2003). "A proximate perspective on reciprocal altruism," Human Nature, ISSN 1045-6767, 13: 129-152.
- Brosnan, S.F., Earley, R.L. and Dugatkin, L.A. (2003). "Observational learning and predator inspection in

guppies (Poecilia reticulata),” Ethology, ISSN 0179-1613, 109: 823-833.
- Caro, T. (1994). Cheetahs of the Serengeti Plains: Group Living in an Asocial Species, ISBN 9780226094342.
- Chapais, B. (1992). “The role of alliances in social inheritance of rank among female primates,” in A. Harcourt and F.B.M. de Waal (eds.), Coalitions and Alliances in Humans and Other Animals, ISBN 9780198542735, pp. 29-59.
- Courchamp, F. and Macdonald, D.W. (2001). “Crucial importance of pack size in the African wild dog Lycaon pictus,” Animal Conservation, ISSN 1367-9430, 4: 169-174.
- Creel, S. and Creel, N.M. (2002). The African Wild Dog: Behavior, Ecology, and Conservation, ISBN 9780691016559.
- Crespi, B. (2001). “The evolution of social behavior in microorganisms,” Trends in Ecology & Evolution, ISSN 0169-5347, 16: 178-183.
- Dawes, R. and Messick, M. (2000). “Social dilemmas,” International Journal of Psychology, ISSN 0020-7594, 35: 111-116.
- de Waal, F.B.M. (1996). Good Natured: The Origins of Right and Wrong in Humans and Other Animals, ISBN 9780674356610.
- de Waal, F.B.M. (1982). Chimpanzee Politics: Power and Sex among Apes, ISBN 9780801863363.
- Doebeli, M. and Hauert, C. (2005). “Models of cooperation based on the Prisoner’s Dilemma and the Snowdrift game,” Ecology Letters, ISSN 1461-023X, 8: 748-766.
- Dugatkin, L.A. (1997). Cooperation Among Animals: An Evolutionary Perspective, ISBN 9780195086225.
- Dugatkin, L.A. (1992). “Tendency to inspect predators predicts mortality risk in the guppy, Poecilia reticulate,” Behav. Ecol., ISSN 1045-2249, 3: 124-128.
- Dugatkin, L.A. (1991). “Dynamics of the TIT FOR TAT strategy during predator inspection in the guppy (Poecilia reticulata),” Behavioral Ecology and Sociobiology, ISSN 0340-5443, 29: 127-132.
- Faaborg, J., Parker, P.G., Delay, L., De Vries, T., Bed- narz, J.C., Paz, S.M., Naranjo, J. and Waite, T.A. (1995). “Confirmation of cooperative polyandry in the Galapagos hawk (Buteo galapagoensis),” Behavioral Ecology and Sociobiology, ISSN 0340-5443, 36: 83-90.
- Faloutsos, M., Faloutsos, P. and Faloutsos, C. (1999). “On power-law relationships of the Internet topology,” in Applications, Technologies, Architectures, and Protocols for Computer Communication: Proceedings of the Conference on Applications, Technologies, Architectures, and Protocols for Computer Communication, ISBN 1581131356, pp. 251-262.
- Fehr, E. and Fischbacher, U. (2003). “The nature of human altruism,” Nature, ISSN 0028-0836, 425: 785-791.
- Fentress, J.C. and Ryon, J. (1986). “A multidimensional approach to agonistic behavior in wolves,” in H. Frank (ed.), Man and Wolf: Advances, Issues and Problems in Captive Wolf Research, ISBN 9789061936145, pp. 253-273.
- Fu, F., Liu, L.-H. and Wang, L. (2007). “Evolutionary Prisoner’s Dilemma on heterogeneous Newman-Watts small-world network,” European Physical Journal B, ISSN 1434-6028, 56: 367-372.
- Gintis, H. (2000). Game Theory Evolving, ISBN 9780691009438.
- Gotts, N.M., Polhill, J.G. and Law, A.N.R. (2003). “Agent-based simulation in the study of social dilemmas,” Artificial Intelligence Review, ISSN 0269-2821, 19: 3-92.
- Hamilton, W.D. (1967). “Extraordinary sex ratios,” Science, ISSN 0036-8075, 156: 477-488.
- Hammerstein, P. (2003). Genetic and Cultural Evolution of Cooperation, ISBN 9780262083263.
- Hanaki, N., Peterhansl, A., Dodds, P.S. and Watts, D.J. (2007). “Cooperation in evolving social networks,” Management Science, ISSN 0025-1909, 53: 1036-1050.
- Harcourt, A. and de Waal F.B.M. (eds.) (1992). Coalitions and Alliances in Humans and Other Animals, ISBN 9780198542735.
- Hofbauer, J. and Sigmund, K. (1998). Evolutionary Games and Population Dynamics, ISBN

9780521625708.
- Huberman, B.A. and Glance, N.S. (1993). “Evolutionary games and computer simulations,” Proceedings of the National Academy of Science, ISSN 0027-8424, 90(16): 7716-7718.
- Jeong, H., Tombor, B., Albert, R., Oltavi, Z. and Barabasi, A. (2000). “The large-scale organization of metabolic networks,” Nature, ISSN 0028-0836, 407:651-654.
- Johnson, D., Stopka, P. and Knights, S. (2003). “The puzzle of human cooperation,” Nature, ISSN 0028-0836, 421: 911-912.
- Lieberman, E., Hauert, C. and Nowak, M.A. (2005). “Evolutionary dynamics on graphs,” Nature, ISSN 0028-0836, 433: 312-316.
- Maynard-Smith, J. and Price, G.R. (1973). “The logic of animal conflict,” Nature, ISSN 0028-0836, 246: 15-18.
- Maynard-Smith, J. (1982). Evolution and the Theory of Games, ISBN 9780521288842.
- Milinski, M. (1987). “Tit for tat in sticklebacks and the evolution of cooperation,” Nature, ISSN 0028-0836, 325: 433-435.
- Nowak, M.A. and May, R.M. (1992). “Evolutionary games and spatial chaos,” Nature, ISSN 0028-0836, 359: 826-829.
- Nowak, M.A. and Sigmund, K. (1992). “Tit for tat in heterogeneous populations,” Nature, ISSN 0028-0836, 355: 250-253.
- Ohtsuki, H., Hauert, C., Lieberman, E. and Nowak, M. (2006). “A simple rule for the evolution of cooperation on graphs and social networks,” Nature, ISSN 0028-0836, 441: 502-505.
- Ostrom, E., Burger, J., Field, C.B., Norgaard, R.B. and Policansky, D. (1999). “Revisiting the commons: Local lessons, global challenges,” Science, ISSN 0036-8075, 284: 278-282.
- Packer, C. and Pusey, A.E. (1982). “Cooperation and competition within coalitions of male lions: Kin selection or game theory?” Nature, ISSN 0028-0836, 296: 740-742.
- Putnam, R.D. (2001). Bowling Alone: The Collapse and Revival of American Community, ISBN 9780743203043.
- Santos, F.C., Pacheco, J.M. and Lenaerts, T. (2006a). “Cooperation prevails when individuals adjust their social ties,” PLoS Computational Biology, ISSN 1553-734X, 2: 1284-1291.
- Santos, F.C., Pacheco, J.M. and Lenaerts, T. (2006b). “Evolutionary dynamics of social dilemmas in structured heterogeneous populations,” Proceedings of the National Academy of Science, ISSN 0027-8424, 103: 3490-3494.
- Santos, F.C., Rodrigues, J.F. and Pacheco, J.M. (2005). “Graph topology plays a determinant role in the evolution of cooperation,” Proceedings of the Royal Society B: Biological Sciences, ISSN 0962-8452, 273: 51-55.
- Santos, F.C. and Pacheco, J.M. (2005). “Scale-free networks provide a unifying framework for the emergence of cooperation,” Physical Review Letters, ISSN 0031-9007, 95: 098104.
- Schelling, T.C. (1978). Micromotives and Macrobehavior, ISBN 9780393090093.
- Skyrms, B. (2003). The Stag Hunt and the Evolution of Social Structure, ISBN 9780521533928.
- Skyrms, B. (1996). Evolution of the Social Contract, ISBN 9780521555838.
- Skyrms, B. and Pemantle, R. (2000). “A dynamic model of social network formation,” Proceedings of the National Academy of Science, ISSN 0027-8424, 97: 9340-9346.
- Strogatz, S. H. (2001). “Exploring complex networks,” Nature, ISSN 0028-0836, 410: 268-276.
- Sugden, R. (1986). The Economics of Rights, Cooperation and Welfare, ISBN 9780631144496.
- Szabo, G. and Fath, G. (2006). “Evolutionary games on graphs,” Unpublished manuscript.
- Tomassini, M., Luthi, L. and Pestelacci, E. (2006). “Social dilemmas and cooperation in complex networks,” Unpublished manuscript.
- Trivers, R.L. (1971). “The evolution of reciprocal altruism,” The Quarterly Review ofBiology, ISSN 0033-5770, 46: 34-57.
- Velicer, G. J. (2005). “Evolution of cooperation: Does selfishness restraint lie within?” Current Biology,

ISSN 0960-9822, 15: 173-175.
- Velicer, G. J. (2003). “Social strife in the microbial world,” Trends in Microbiology, ISSN 0966-842X, 11: 330-337.
- Velicer, G.J., Kroos, L. and Lenski, R.E. (2000). “Developmental cheating in the social bacterium Myxococcus Xanthus,” Nature, ISSN 0028-0836, 404: 598-601.
- Velicer, G.J. and Stredwick, K.L. (2002). “Experimental social evolution with Myxococcus Xanthus,” Antonie van Leeuwenhoek, ISSN 0003-6072, 81: 155-164.
- Von Neumann, J. and Morgenstern, O. (1944). Theory of Games and Economic Behavior, ISBN 9780691003627.
- Williams, R.J. and Martinez, N.D. (2000). “Simple rules yield complex food webs,” Nature, ISSN 0028-0836, 404: 180-183.
- Wilson, E.O. (2000). Sociobiology: The New Synthesis, Twenty-fifth Anniversary Edition, ISBN 9780674002357.
- Wilson, E.O. (1998). Consilience: The Unity of Knowledge, ISBN 9780679768678.